

# Pre-Calculus: Polynomial & Rational Functions

## Stage 1 Desired Results

<p><b>ESTABLISHED GOALS:</b></p> <p><u>Competencies:</u></p> <ul style="list-style-type: none"> <li>Students will demonstrate the ability to simplify algebraic expressions by applying the properties of operations and arithmetic of complex numbers.</li> <li>Students will demonstrate the ability to solve equations, inequalities and systems by analyzing structure and applying the properties of equality, inequality, and rational expressions.</li> <li>Students will demonstrate the ability to apply functions to solve problems by interpreting and analyzing multiple representations of functions.</li> <li>Students will demonstrate the ability to graph equations, functions, and figures by using tables and analyzing equations.</li> <li>Students will demonstrate the ability to model real world problems by building and analyzing the appropriate expression, equation, or function.</li> <li>Students will demonstrate the ability to analyze and summarize text and integrate knowledge to make meaning of discipline-specific materials.</li> <li>Students will demonstrate the ability to produce coherent and supported writing in order to communicate effectively for a range of discipline-specific tasks, purposes, and audiences.</li> <li>Students will demonstrate the ability to speak purposefully and effectively by strategically making decisions about content, language use, and discourse style.</li> </ul> <p><u>Content Standards:</u></p> <ul style="list-style-type: none"> <li>HSN.CN.A.1 Know there is a complex number <math>i</math> such that <math>i^2 = -1</math>, and every complex number has the form <math>a + bi</math> with <math>a</math> and <math>b</math> real.</li> <li>HSN.CN.A.2 Use the relation <math>i^2 = -1</math> and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.</li> <li>HSN.CN.A.3 (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.</li> <li>HSN.CN.B.4 (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.</li> <li>HSN.CN.B.5 (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation.</li> <li>HSN.CN.B.6 (+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.</li> </ul>	<b>Transfer</b>	
	<p><i>Students will be able to independently use their learning to represent relationships between quantities, manipulate and analyze the representation, and interpret its meaning.</i></p>	
	<b>Meaning</b>	
	<p><b>ENDURING UNDERSTANDINGS</b> <i>Students will understand that...</i></p> <ul style="list-style-type: none"> <li>equivalent expressions can be created by using multiple strategies and the guiding rules of mathematics.</li> <li>the graph and equation of a function communicate information that can be used to answer questions about real world problems.</li> <li>it is possible to get closer and closer to something but to never be able to touch it.</li> </ul>	<p><b>ESSENTIAL QUESTIONS</b></p> <ul style="list-style-type: none"> <li>How can the multiple representations of a function be used to best analyze the relationships between the two quantities it models?</li> </ul>
<b>Acquisition</b>		
<p><i>Students will know...</i></p> <ul style="list-style-type: none"> <li>the average rate of change of a function <math>f</math> on the interval <math>[a, b]</math> is the slope of the line joining two points <math>(a, f(a))</math> and <math>(b, f(b))</math>.</li> <li>that a graph of a function can be sketched by identifying the parent function and then analyzing and interpreting the changes made to <math>f(x)</math>, for example <math>-f(x)</math> or <math>f(x+c)</math>.</li> <li>that given <math>f(x)=x^n</math>, when <math>n</math> is even, the graph of the function is symmetric with respect to the <math>y</math>-axis, and when <math>n</math> is odd, it is symmetric with respect to the origin.</li> <li>that, if the multiplicity of a polynomial's zero is odd, the graph crosses the <math>x</math>-axis at that value. If it is even, then it touches, but does not cross, the <math>x</math>-axis at that value.</li> <li>that the graph of a polynomial function of degree 2 or greater is a continuous smooth</li> </ul>	<p><i>Students will be skilled at...</i></p> <ul style="list-style-type: none"> <li>calculating the instantaneous rate of change using the average rate of change.</li> <li>computing, interpreting, and comparing average rate of change for a function.</li> <li>simplifying difference quotients of the form <math>f(x)-f(a) \div x-a</math>.</li> <li>determining the domain and range of a function from both an equation and from a graph.</li> <li>graphing the parent function and transformations for linear, quadratic, square root, and absolute value functions.</li> <li>graphing transformations by viewing them as changes made to the parent function.</li> <li>setting up equations that define functions for problem-solving, including maximum and minimum value problems.</li> </ul>	

<ul style="list-style-type: none"> <li>• HSN.CN.C.8 (+) Extend polynomial identities to the complex numbers. For example, rewrite <math>x^2 + 4</math> as <math>(x + 2i)(x - 2i)</math>.</li> <li>• HSN.CN.C.9 (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.</li> <li>• HSA.APR.A.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.</li> <li>• HSA.APR.B.2 Know and apply the Remainder Theorem: For a polynomial <math>p(x)</math> and a number <math>a</math>, the remainder on division by <math>x - a</math> is <math>p(a)</math>, so <math>p(a) = 0</math> if and only if <math>(x - a)</math> is a factor of <math>p(x)</math>.</li> <li>• HSA.APR.B.3 Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.</li> <li>• HSA.APR.C.4 Prove polynomial identities and use them to describe numerical relationships. For example, the polynomial identity <math>(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2</math> can be used to generate Pythagorean triples.</li> <li>• HSA.APR.C.5 (+) Know and apply the Binomial Theorem for the expansion of <math>(x + y)^n</math> in powers of <math>x</math> and <math>y</math> for a positive integer <math>n</math>, where <math>x</math> and <math>y</math> are any numbers, with coefficients determined for example by Pascal's Triangle.1</li> <li>• HSA.APR.D.6 Rewrite simple rational expressions in different forms; write <math>a(x)/b(x)</math> in the form <math>q(x) + r(x)/b(x)</math>, where <math>a(x)</math>, <math>b(x)</math>, <math>q(x)</math>, and <math>r(x)</math> are polynomials with the degree of <math>r(x)</math> less than the degree of <math>b(x)</math>, using inspection, long division, or, for the more complicated examples, a computer algebra system</li> <li>• HSA.APR.D.7 (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.</li> <li>• HSA.REI.D.11 Explain why the <math>x</math>-coordinates of the points where the graphs of the equations <math>y = f(x)</math> and <math>y = g(x)</math> intersect are the solutions of the equation <math>f(x) = g(x)</math>; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where <math>f(x)</math> and/or <math>g(x)</math> are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.</li> <li>• HSF.IF.B.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.</li> <li>• HSF.IF.B.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.*</li> <li>• HSF.IF.C.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.</li> <li>• HSF.IF.C.7.C Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.</li> <li>• HSF.IF.C.7.D (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing</li> </ul>	<p><i>curve.</i></p> <ul style="list-style-type: none"> <li>• <i>leading Coefficient Test for polynomial functions.</i></li> <li>• <i>that, for a polynomial function of degree, <math>n</math> has at most, <math>n</math> real zeros.</i></li> <li>• <i>that the graph of a polynomial function of degree <math>n</math> has at most <math>n-1</math> turning points.</i></li> <li>• <i>the relationships between roots and coefficients for polynomials equations of any degree.</i></li> <li>• <i>that complex zeros occur in conjugate pairs.</i></li> <li>• <i>Descartes's Rule of Sign.</i></li> <li>• <i>that, if the degree of the numerator is exactly one more than the degree of the denominator, then the graph of the function has a slant asymptote.</i></li> <li>• <i>that, within an interval of the number line created by the <math>x</math>-intercepts, the sign of the value of the function stays the same.</i></li> <li>• <i>the following algorithms and theorems:</i> <ul style="list-style-type: none"> <li>○ <i>Division Algorithm</i></li> <li>○ <i>Remainder Theorem</i></li> <li>○ <i>Intermediate Value Theorem</i></li> <li>○ <i>Factor Theorem</i></li> <li>○ <i>Fundamental Theorem of Algebra</i></li> <li>○ <i>Linear Factorization Theorem</i></li> <li>○ <i>Rational Roots Theorem</i></li> <li>○ <i>Upper and Lower Bound Theorem for</i></li> <li>○ <i>Upper and Lower Bound Theorem for Real Roots.</i></li> </ul> </li> </ul> <p><u><i>vocabulary:</i></u> <i>rational function, key numbers, vertical/horizontal/slant asymptote, complex conjugate, double root, extraneous roots, persistence of sign, , synthetic division, even and odd functions</i></p>	<ul style="list-style-type: none"> <li>• <i>sketching polynomial functions, using the leading coefficient test, the real zeros, and test intervals.</i></li> <li>• <i>using the Intermediate Value Theorem to approximate a real zero.</i></li> <li>• <i>sketching rational functions.</i></li> <li>• <i>using long and synthetic division to find quotients and remainders and to show that a value of <math>x</math> is a solution to the equation.</i></li> <li>• <i>factoring polynomials.</i></li> <li>• <i>finding solutions to polynomial equations using the Factor Theorem.</i></li> <li>• <i>simplifying rational expressions by factoring and by using long/synthetic division.</i></li> <li>• <i>adding, subtracting, and multiplying complex numbers.</i></li> <li>• <i>determining the quotient of complex numbers in standard form.</i></li> <li>• <i>writing complex numbers in standard form</i></li> <li>• <i>expressing polynomial equations in the form <math>a_n(x-r_1)(x-r_2)...(x-r_n)</math></i></li> <li>• <i>determining a polynomial equation with prescribed roots and their multiplicities.</i></li> <li>• <i>finding the rational roots of a polynomial equation.</i></li> <li>• <i>solving an equation using the rational roots theorem and the upper and lower bound theorem.</i></li> <li>• <i>solving an equation using the conjugate roots theorem.</i></li> <li>• <i>determining remaining roots when provided one root.</i></li> <li>• <i>using Descartes's Rule to verify the number of positive/negative roots.</i></li> <li>• <i>determining the vertical and horizontal asymptotes of a rational function.</i></li> <li>• <i>sketching the graph of rational functions.</i></li> <li>• <i>finding the equation of a slant asymptote.</i></li> <li>• <i>solving polynomial inequalities using factoring, key numbers, and the persistence of sign.</i></li> </ul>
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<ul style="list-style-type: none"> <li>end behavior.</li> <li>MP1 Make sense of problems and persevere in solving them.</li> <li>MP2 Reason abstractly and quantitatively.</li> <li>MP4 Model with mathematics.</li> <li>MP5 Use appropriate tools strategically.</li> <li>MP7 Look for and make use of structure.</li> <li>MP8 Look for and express regularity in repeated reasoning.</li> </ul>		
<b>Content Area Literacy Standards</b>		<b>21<sup>st</sup> Century Skills</b>
<ul style="list-style-type: none"> <li>RST.11-12.3 Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks; analyze the specific results based on explanations in the text.</li> <li>RST.11-12.4 Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to <i>grades 11-12 texts and topics</i>.</li> <li>RST.11-12.5 Analyze how the text structures information or ideas into categories or hierarchies, demonstrating understanding of the information or ideas.</li> <li>RST.11-12.8 Evaluate the hypotheses, data, analysis, and conclusions in a science or technical text, verifying the data when possible and corroborating or challenging conclusions with other sources of information.</li> </ul>		<ul style="list-style-type: none"> <li><i>reason effectively</i></li> <li><i>use systems thinking</i></li> <li><i>solve problems</i></li> <li><i>apply technology effectively</i></li> </ul>

<b>Stage 2 - Evidence</b>	
<b>Evaluative Criteria</b>	<b>Assessment Evidence</b>
	PERFORMANCE TASK(S):
	OTHER EVIDENCE:

<b>Stage 3 – Learning Plan</b>	
<i>Summary of Key Learning Events and Instruction</i>	
<b>Language Arts Integration</b>	<b>Mathematics Integration</b>
<ul style="list-style-type: none"> <li>1.OA.1 Use</li> </ul>	<ul style="list-style-type: none"> <li>1.OA.1 Use</li> </ul>

<b><i>Technology Integration</i></b>	<b><i>District Materials</i></b>
<ul style="list-style-type: none"><li>• 1.OA.1 Use</li></ul>	